STRESSES

Stresses due to internal pressure are a major part in pressure vessels. The tensile stress in thin (R/t > 10) shells due to pressure is considered to be uniform across thickness and is called membrane stress.

Due to internal pressure, the element of vessels is subjected to three principal stresses:

1. *Circumferential/hop*: This is approximately equal to PR/t, maximum on the inside surface, and minimum on the outside surface.
2. *Longitudinal*: This is approximately equal to $PR/2t$, maximum on the inside surface, and minimum on the outside surface.
3. *Radial*: This is equal to design pressure on the inside surface, and zero on the outside surface.

The difference of stress from inside to outside decreases with the increase in the R/t ratio and is insignificant above R/t ratio of 10. The radial stress above this ratio is insignificant compared to other two stresses and can be neglected. Codes give a slightly modified formula for stresses (average) to cover most of the practical shells up to *R/t* = 2. Almost all applications, the *R/t* ratio of shells will be above this value.

The shell thickness computed by the Code formulas for internal or external pressure alone is often not sufficient to withstand the combined effects of all other loadings. Detailed calculations consider the effects of each loading separately and then must be combined to give the total state of stress in that part.

Types of stress, stress categories, and allowable stresses are based on the type of loading that produced them and on the hazard they represent to the structure. Unrelenting loads produce primary stresses. Relenting loads (self-limiting) produce secondary stresses. Primary stresses must be kept lower than secondary stresses. Primary plus secondary stresses are allowed to be higher and so on. Before considering the combination of stress categories, we must first define the various types of stress and each category.

The following list of stresses describes types of stresses without regard to their effect on the vessel or component.

Tensile	Bearing	Load controlled
Compressive	Axial	Strain controlled
Normal	Discontinuity	Circumferential
Shear	Principal	Longitudinal
Membrane	Thermal	Radial
Bending	Tangential	

STRESS CATEGORIES

The foregoing list provides the categories and subcategories. Stress categories are defined by the type of loading which produces them and the hazard they represent to the vessel.

1) *Primary Stress*
   a. General membrane stress, P_m
   b. Local membrane stress, P_L
   c. Bending stress, P_b
2) *Secondary Stress*
   a. Secondary membrane stress, Q_m
   b. Secondary bending stress, Q_b
3) *Peak Stress*, F

The primary stress is related to mechanical loading directly and satisfies force and moment equilibrium. Primary stress that exceeds the yield stress will result in failure. By contrast, secondary stresses are those arising from geometric discontinuities or stress concentrations. For an increasing external load, at any point, both primary and secondary stresses increase in proportion to this load, until the yield point is reached. But secondary stresses are termed self-limiting; that is, once the yield point has been passed locally around the stress concentration, the direct relationship between load and stress is broken, due to the reduced post-yield stiffness of the material. This contrasts with primaries (sometimes called "load-controlled" stresses) that will continue to increase in overall magnitude, in direct proportion to the applied loads, irrespective of the shape of the stress-strain curve, until failure.

In a region away from any discontinuities, only primary stress will arise. Secondary stress cannot arise alone however – at a discontinuity, the secondary stress will be superimposed on the underlying primary stress.

Peak stress is the highest stress condition in a structure and is usually due to stress concentration caused by an abrupt change in geometry. This stress is important in considering fatigue failure because of cyclic load application.

PRIMARY STRESSES

Primary stresses are normal or shear stresses which are required to satisfy equilibrium. They are produced by mechanical loads (load controlled), such as pressure load, and when exceeding the yield strength can result in failure or gross distortion. The basic characteristic of a primary stress is that it is not self-limiting. Thermal stresses from thermal gradients and imposed displacements are never classified as primary stresses.

Primary stresses cause ductile rupture or a complete loss of load-carrying capability due to plastic collapse of the structure upon a single application of load. The purpose of the code limits on primary stress is to prevent gross plastic deformation and to provide a nominal factor of safety on the ductile burst pressure.

Primary general stresses are divided into membrane and bending stresses. The need for this division is that the calculated value of a primary bending stress may be allowed to go to a higher value than that of a primary general membrane stress.

1) *Primary general*: Primary general stresses are generally due to internal or external pressure or produced by sustained external forces and moments.

   a. *Primary general membrane stress*, P_m: Primary stress is the average primary stress across a solid section; it is produced by pressure or mechanical loads; and it is remote from discontinuities such as head-shell intersections, cone-cylinder intersections, nozzles, and supports. General primary membrane stress is so distributed in the structure that no redistribution of the load occurs as result of yielding. Examples are:

      - Shells away from discontinuities due to internal pressure.
      - Compressive and tensile axial stresses due to wind.
      - Axial compression due to weight.
      - Nozzles within the limits of reinforcement due to internal pressure.

   b. *Primary general bending stress*, P_b: Primary bending is the component of primary stress proportional to the distance from the centroid of the solid section and is produced by mechanical loads. The membrane stress is the component having a constant value through the section and represents an average value. These stresses are due to pressure or mechanical loads and are through thickness. These stresses are due to sustained loads and are capable of causing collapse of vessels. There are relatively few areas where primary bending occurs:

      - Center of flat head or crown of a dished head.
      - Shallow conical head

- In the ligaments of closely spaced openings

2) *Primary local membrane stress, PL:* A primary local membrane stress is produced by sustained loads - either by design pressure alone or by other mechanical loads and are limited to a distance of $\sqrt{Rt}$ in meridian direction. These stresses have self-limiting characteristics like secondary stresses. Since they are localized, once the yield strength of the material is reached, the load is redistributed to stiffer portions of the vessel. However, since any deformation associated with yielding would be unacceptable, an allowable stress lower than secondary stress is assigned. The ability of primary local membrane stresses to redistribute after the material yields allows for a higher allowable stress but only in the local area.

   Quite often the concepts of general primary membrane stress and local primary membrane stress are used interchangeably; the local primary membrane stress representing a general primary membrane stress along a local structural discontinuity. The local primary membrane stress is the average across a solid section that includes discontinuities.

   The bending stresses associated with local loading are almost always classified as secondary stresses. Therefore, the membrane stresses from a WRC-107 type analysis must be broken out separately and combined with general primary stresses due to internal pressure, for example.

   Examples of primary local membrane stress are:

   - Where internal pressure is the origin of stress and it is at a discontinuity,
     - On the shell near a nozzle or other opening
     - Head-shell juncture
     - Cone-cylinder juncture
     - Shell-flange juncture
     - Head-skirt juncture
     - Shell-stiffening ring juncture
   - Where non-pressure applied loads are the origin of stress and they are at a discontinuity,
     - Support lugs
     - Nozzle external loads
     - Beam supports
     - Major attachments

SECONDARY STRESSES

Secondary stresses arise from geometric discontinuities or stress concentrations. They are normal or shear stresses which are required to satisfy the imposed strain or displacement (continuity requirement) as opposed to being in equilibrium with the external load.

The basic characteristic of a secondary stress is that it is self-limiting. This means that local yielding and minor distortions can satisfy the conditions which caused the stress to occur. Application of secondary stress cannot cause structural failure of the vessel due to the restraint offered by the body to which the part is attached. Secondary stress can develop at structural discontinuities but is also used to describe through-thickness gradients away from structural discontinuities. Secondary stresses are also produced by sustained loads other than internal or external pressure.

Structural discontinuities that develop secondary stresses should be placed apart by at least $2.5\sqrt{R_m t}$. This restriction is to eliminate the additive effects of edge moments and forces.

Secondary stresses are divided into two additional groups – membrane and bending.

1. *Secondary membrane stress*, Q_m:

   Examples are,

- Axial thermal gradients in shells, cones, or formed heads.
- Thermal gradients between the shell and head.
- Thermal stress due to differential thermal expansion within a nozzle wall.
- Pressure stress at an isolated ligament.

2. *Secondary bending stress*, Q_b:

   Examples are,

   - Axial thermal gradients in shells, cones, or formed heads.
   - Thermal gradients between the shell and head.
   - Head-shell juncture.
   - Nozzles outside the limits of reinforcement due to pressure and external loading.
   - Thermal stress due to differential thermal expansion within a nozzle wall.

PEAK STRESS, F

Peak stresses are additive to primary and secondary stresses present at the point of stress concentration (such as a notch or weld discontinuity). They can also be produced by certain thermal stress. Peak stress does not cause significant distortion but may cause fatigue failure.

Peak stresses apply to both sustained loads and self-limiting loads. They are only significant in fatigue conditions or brittle materials. Peak stresses are sources of fatigue cracks and apply to normal and shear stresses. Examples are:

- Stresses at corner of a discontinuity (e.g., fillet weld or corner).
- Thermal stresses due to differential thermal expansion within a nozzle wall.
- Thermal stresses in a cladding or wall overlay.
- stress due to the notch effect (stress concentration).

STRESS CLASSIFICATIONS

The stress classifications for various parts of a pressure vessel are indicated in Table 1. It can be observed that the membrane stress is considered primary for mechanical loads. For a number of geometries and loading situations, the bending stress is considered secondary. The bending stress is considered primary when the net section experiences the applied bending moment.

Table 1: Classification of Stresses

Vessel Part	Location	Origin of Stress	Type of Stress	Classification
Cylindrical or spherical shell	Remote from discontinuities	Internal pressure	General membrane	P_m
			Through thickness gradient	Q
	Junction with head or flange	Axial thermal gradient	Membrane	Q
			Bending	Q
Any shell or head	Any section across entire vessel	External load or moment, or internal pressure	General membrane averaged across full section	P_m
		External load or moment	Bending across full section	P_m
	Near nozzle or other opening	External load or moment, or internal pressure	Local membrane	P_L
			Bending	Q

			Peak (fillet or corner)	F
	Any location	Temperature difference between shell and head	Membrane	Q
			Bending	Q
Dished head or conical head	Crown	Internal pressure	Membrane	P_m
			Bending	P_b
	Knuckle or junction to shell	Internal pressure	Membrane	P_L
			Bending	Q
Flat head	Center region	Internal pressure	Membrane	P_m
			Bending	P_b
	Junction to shell	Internal pressure	Membrane	P_L
			Bending	Q

STRESS LIMITS

Potential failure modes and the various stress limits categories are related. Limits on primary stresses are set to prevent deformation and ductile burst. The primary plus secondary limits are set to prevent plastic deformation leading to incremental collapse and to validate using an elastic analysis to perform fatigue analysis. Finally, peak stress limits are set to prevent fatigue failure due to cyclic loadings.

Table 2 below is derived from ASME VIII-2. It should be used as a guide only because ASME VIII-1 recognizes only two categories of stresses – primary membrane stress and primary bending stress. In addition, ASME VIII-2 utilizes load combinations, by which short term loads (such as seismic) are reduced when combined with other loads. It also sets the allowable limits of combined stresses for fatigue loading where secondary and peak stresses are major considerations.

Table 2: Allowable Stresses for Stress Classifications and Categories

Stress Classification or Category	Allowable Stress
General primary membrane, P_m	SE
Primary membrane stress PLUS primary bending stress across the thickness, $P_m + P_b$	1.5SE
Local primary membrane, P_L	1.5SE
Local primary membrane PLUS primary bending, $P_L + P_b$	1.5SE
Secondary membrane PLUS secondary bending, $Q_m + Q_b$	$3SE < 2F_y$
P + Q	$3SE < 2F_y$
$P_m + P_b + Q_m + Q_b$	$3SE < 2F_y$
$P_L + P_b + Q_m + Q_b$	$3SE < 2F_y$
Peak, F	S_a
P + Q + F	S_a
$P_m + P_b + Q_m + Q_b + F$	S_a

$P_L + P_b + Q_m + Q_b + F$	S_a

Notes:

F_y = minimum specified yield strength at design temperature

E = joint efficiency

S = allowable stress at design temperature

S_a = alternating stress for any given number of cycles from design fatigue curves

The term 3SE shall be used in lieu of $2F_y$ when the ratio of minimum specified yield strength to ultimate strength exceeds 0.7 or S is governed by time-dependent properties

THERMAL STRESSES

Thermal stresses are developed whenever the expansion or contraction that would occur normally as a result of heating or cooling of components is prevented. These are "secondary stresses" because they are self-limiting. That is, yielding or deformation of the part relaxes the stress. Thermal stress will not cause failure by rupture in ductile material except by fatigue over repeated application. They can, however, cause failure due to excessive deformations.

Prevention of expansion or contraction, i.e., mechanical restraints are either internal or external.

External restraint occurs when an object or a component is supported or contained in a manner that restricts thermal movement. An example of external restraint occurs when piping expands into a vessel nozzle creating a radial load on the vessel shell.

Internal restraint occurs when the temperature through an object is not uniform. Stresses from a "thermal gradient" are due to internal restraint. Stress is caused by a thermal gradient whenever the temperature distribution or variation within a member creates a differential expansion such that the natural growth of one fiber is influenced by the different growth requirements of adjacent fibers. The result is distortion or warpage.

A transient thermal gradient occurs during heat-up and cool-down cycles where the thermal gradient is changing with time. Thermal gradients may be logarithmic or linear across a vessel wall. Given a steady heat input inside or outside a tube, the heat distribution will be logarithmic if there is a temperature difference between the inside and outside the tube. This effect is significant for thick-walled vessels. A linear temperature distribution may be assumed if the wall is thin. Stress calculations are much simpler for linear distribution.

Difference Between Mechanical Stress and Thermal Stress

The fundamental difference between mechanical stresses and thermal stresses lies in the nature of the loading. Thermal stresses are a result of restraint or temperature distribution. The stress pattern must only satisfy the requirements for equilibrium of the internal forces. The result being that yielding will relax the thermal stress. If the part is loaded mechanically beyond its yield strength, the part will continue to yield until it breaks. The external load remains constant, thus the internal stresses cannot relax.

Almost all equipment (except piping) are rigid between supports. Therefore, one support is designed as fixed and other sliding. If both supports are fixed, expansion joint (bellow) is provided between them. In such case, pressure thrust is induced in both fixed supports and generally not viable to design support. For low pressures, ties are provided between two parts separated by expansion joint to prevent supports from getting pressure thrust.

Piping systems are generally not rigid and providing one fixed and rest sliding will land in vibration problems. Several supports or restraints are required to prevent vibration. Pressure thrust is not high, and design of support or restraint is viable. A system can be designed to provide sufficient flexibility. Bellows are provided for low pressures and only when the required flexibility cannot be provided.

In general, thermal stresses are considered only in secondary and peak categories. Thermal stresses that cause a distortion of the structure are categorized as secondary stresses; thermal stresses caused by suppression of thermal expansion, but that may not cause distortion, are categorized as peak stresses.

DISCONTINUITY STRESSES

Vessel sections of different thickness, material, diameter, and change in directions would all have different displacements if allowed to expand freely. However, since they are connected in a continuous structure, they must deflect and rotate together. The stresses in the respective parts at or near the juncture are called discontinuity stresses. Discontinuity stresses are local in extent but can be of very high magnitude. They are self-limiting but some stresses require to be classified as local primary membrane stresses to avoid distortion. That is, once the structure has yielded, the forces causing excessive stresses are reduced. In a typical application they will not lead to failure. Discontinuity stresses become an important factor in fatigue design where cyclic loading is a consideration. Design of juncture of two parts is a major consideration in reducing discontinuity stresses.

It is necessary to superimpose the general membrane stresses with discontinuity stresses. From superimposition of these two states of stress. The total stresses are obtained. Generally, when combined, a higher allowable stress is permitted. The designer should be aware that for designs of high pressure (>1500 psi), brittle material, or cyclic loading, discontinuity stresses may be a major consideration.

REFERENCE:

Pressure Vessel Design Manual, *by* Dennis Moss and Michael Basic

Structural Analysis and Design of Process Equipment. *By* Maan Jawad and James Farr

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ASME Boiler and Pressure Vessel Code, Section VIII, Division 1 and Division 2